NOISE CHARACTERIZATION: SIMULTANEOUS FREQUENCY AND TIME DOMAIN TRANSACTION

A Wavelet Approach
Signal Integrity characterization using wavelets

- Interconnect Model order reduction is done around DC in all the commercial software.
- Noise is a high frequency content, therefore, has to be added at particular scale for delay modification and glitch.
- Since Fourier spectrum is available for free during wavelet characterization, most of the calculations are done in frequency domain.
- Details will appear in another white paper.
Least Square Approximation – Wavelets

- It’s a recursive algorithm for least square approximation with decreasing errors at each stage and finally the error converges to zero in 2-3 steps.
- $\Psi(t)$: is a fixed point solution of refinement equation in Banach space
- A family of series of functional can be generated for a good starting function
- Least square estimation of $f(t)$ in Hilbert space with wavelet basis

$$H_{j,k} = \int_{-\infty}^{\infty} f(t) \Psi^*(t)_{j,k} \, dt$$

$$\int_{-\infty}^{\infty} \Psi(t)_{j,k} \Psi^*(t)_{j,k} \, dt = \delta_{j,k}$$

$$f(t) = \int_{-\infty}^{\infty} H_{j,k} \Psi(t)_{j,k} \, dt$$

- If we can find an Orthogonal Matrix $H$ of inner product coefficients then DONE!
- Otherwise, perform clever manipulation of the matrix coefficients so that the transformation becomes orthogonal while satisfying ALL the underlying properties of $H$, wavelet and scaling function.
- Bi-orthonormal transformation can be done faster using MRA, otherwise a proper interleaving of both forward and backward transformation matrices is required.
Least Square Approximation - Wavelets

- Key is to find a scaling function $\varphi(t)$
- Find ALL functions in $V^0$-space at resolution zero
  \[ S(t) = \sum_{i=0}^{n} s(i)\varphi(t - i) \]
- Find $V^n$-space at resolution $2^n$
  - Build higher spaces starting from $V^1$-space:
  - scaled down by 2 and translated by $\frac{1}{2}$
  - \[ G(t) = \sum_{i=0}^{n} g(i)\varphi(2t - i) \]
- REFINEMENT EQ: \[ \varphi(t) = \sum_{i=0}^{n} r(i)\varphi(2t - i) \]
- GUARANTEED if $r(i)$ can be found?
Least Square Approximation - Wavelets

- Coefficients in $V^1$ - Space for $S(t)$

$$S(t) = \sum_{i=0}^{n} S[i] \varphi(2t - i)$$

- And the exact solution of the function is

$$S(i) = \sum_{j=0}^{n} S[j] h(i - 2j)$$

  - Functions in $V^1$ - Space will have approx. errors if represented in $V^0$ - space. Let's do it anyhow for $G(t)$

- $G(t) = V^0(t) + V^1(t) = \text{LowPass} + \text{HighPass}$

  = Solution at Resolution 0 + Error Details

- Repeat above steps till convergence
Wavelet Creation

- Find a scaling function $\varphi(t)$ using fixed point solution of refinement/dilation equation in Banach space.

- Such that ALL properties of $\varphi(t)$ and $\Psi(t)$ are fully satisfied. Patience needed here.

- Dilation Equation or Refinement Equation is a Linear combination of half-scale translated version:

$$\varphi(t) = \sum_{i=0}^{n} C_i \ast \varphi(2t - i)$$
Dyadic Grid - DWT

- Convert scale and shift parameters as a function of integers \((j, k)\)

\[
S = 2^{-j}; \quad \tau = S \times k; \quad j = 1, 2 \ldots \quad k = 1, 2 \ldots
\]

\[
\Psi_s, \tau(t) = S^{-1/2} \Psi[(t - \tau)/S]
\]

- Continuous axis \((s, \tau)\) is now discrete axis \((j, k)\)

\[
\Psi_{j, k}(t) = 2^{j/2} \Psi(2^{j/2} * t - k)
\]
DWT

- **Forward Transform:** \( \Psi_{j,k}(t) = 2^{j/2} \Psi(2^{j/2}t - k) \)

\[
a_{j,k} = \sum_t f(t) * \Psi^{*}_{j,k}(t)
\]

- **Inverse transform:**

\[
f(t) = \sum_k \sum_j a_{j,k} * \Psi_{j,k}(t)
\]
CWT/DWT Algorithm

Begin at a initial scale and a zero shift.
For (i = 0; i < Total_SCALES; i++) {
    For (j = 0; j < Total_SHIFTS; j++) {
        For (k = 0; k < Total_SAMPLES; k++) {
            ✓ Place wavelet at the beginning of signal
            ✓ Compute modulation of signal
            ✓ The result is one element of H(j, k) Matrix
        }
    }
}

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A simple iteration of *FIXED POINT SOLUTION* in Banach space for Haar wave

\[ \phi(x) = \phi(2x) + \phi(2x - 1) \]
\[ \Psi(x) = \phi(2x) - \phi(2x - 1) \]